

Calculus III Test

Nov 9, 2005

Time allowed: 45 min

Name:

student number:

All questions are multiple-choice. Please enter answers by marking the appropriate box. Blank sheets are provided for rough work.

The maximum mark is 35.

Questions 1–5 carry 3 marks each, and 6–10 carry 4 marks each. Each question left unanswered gets 1 mark. Incorrect answers get no marks.

There are several equivalent but not identical versions of this paper. Do not be concerned if others appear to have a different paper.

1. The expression $\mathbf{r} = \phi^2(\mathbf{i} \cos \phi + \mathbf{j} \sin \phi)$ with $0 \leq \phi \leq \infty$ represents

☐ the x, y plane.

☐ a spiral in the x, y plane.

☐ the circle of radius ϕ^2 in the x, y plane.

2. The expression Δr^2 (where $\Delta = \nabla \cdot \nabla$) equals

☐ 0

☐ 6

☐ $2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

3. $\mathbf{F}$ is a vector field. The expression $\nabla(\mathbf{F} \cdot \nabla)$ is

☐ a vector field.

☐ a vector operator.

☐ a scalar operator.

4. In the Fourier expansion

$$\sin 2x = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

☐ $b_2 = 1$, all others = 0.

☐ $a_0 = 1$, $a_2 = 1$, all others = 0.

☐ $a_0 = 1$, $b_n = n^{-2}$, $a_n = 0$.

5. To extremize

$$\int L(y, y', x) dx$$

one requires

☐ $d/dx (\partial L / \partial y') = \partial L / \partial y$

☐ $\partial L / \partial y' = \text{const}$

☐ $y' (\partial L / \partial y') - L = \text{const}$

Continued overleaf

6. The plane tangent to the surface $x^2 + y^2 = 100$ at the point $(8, 6, 0)$ is

☐ $3x + 4y = 25$.

☐ $4x + 3y = 50$.

☐ $8x + 6y + 5z = 100$.

☐ $\frac{1}{8}x + \frac{1}{6}y = \frac{1}{5}$.

7. Given a constant vector $\mathbf{A}$, the expression

$$\frac{d\hat{\mathbf{r}}}{dt} \times \mathbf{A}$$

equals [where dot denotes d/dt]

☐ 0

☐ $\frac{\dot{\mathbf{r}} \times \mathbf{A}}{r}$

☐ $\frac{\dot{\mathbf{r}} \times \mathbf{A}}{r} + \dot{r} \frac{\mathbf{A} \times \mathbf{r}}{r^2}$

☐ $\frac{\dot{\mathbf{r}} \times \mathbf{A}}{r} + \mathbf{r} \cdot \nabla \mathbf{A}$

8. The vector field in the accompanying figure has

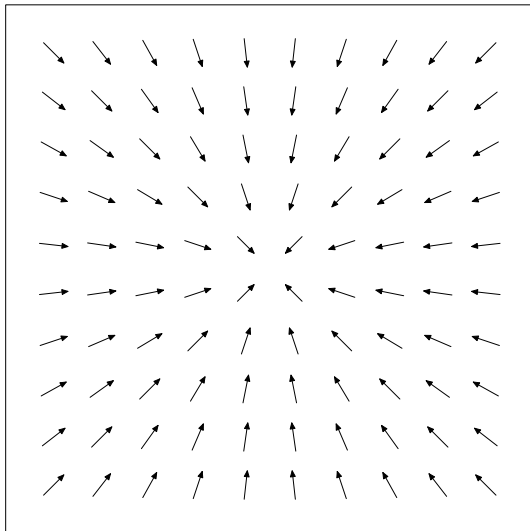


Figure for question 8.

☐ zero divergence and curl.

☐ zero divergence, nonzero curl.

☐ nonzero divergence, zero curl.

☐ nonzero divergence and curl.

9. The curl of $-x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ is

☐ 0

☐ 3

☐ $-\mathbf{i} + \mathbf{j} + \mathbf{k}$

☐ $2\mathbf{k}$

10. To extremize

$$\int (y'^2 + y^2) dx$$

a possible solution is

☐ $y = \sin x$.

☐ $y = \sinh x$.

☐ $y = x$.

☐ $y = \ln x$.

End of test